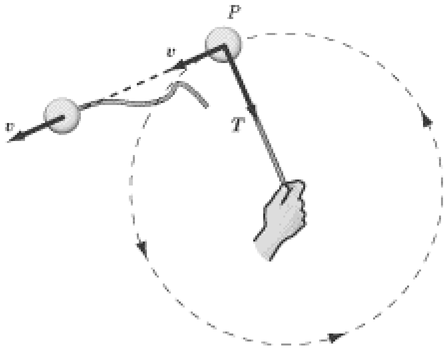
Circular Motion and Gravitation

NEWTON'S FIRST LAW TELLS US THAT objects will move in a straight line at a constant speed unless a net force is acting upon them. That rule would suggest that objects moving in a circle—whether they're tetherballs or planets—are under the constant influence of a changing force, since their trajectory is not in a straight line. We will begin by looking at the general features of circular motion and then move on to examine gravity, one of the principal sources of circular motion.

Uniform Circular Motion

Uniform circular motion occurs when a body moves in a circular path with constant speed. For example, say you swing a tethered ball overhead in a circle:

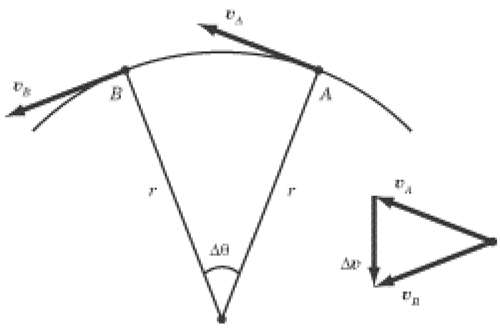


If we leave aside gravity for the moment, the only force acting on the ball is the force of tension, ***T***, of the string. This force is always directed radially inward along the string, toward your hand. In other words, the force acting on a tetherball travelling in a circular path is always directed toward the centre of that circle.

Note that although the direction of the ball's velocity changes, the ball's speed is constant in magnitude and is always tangent to the circle.

Centripetal Acceleration

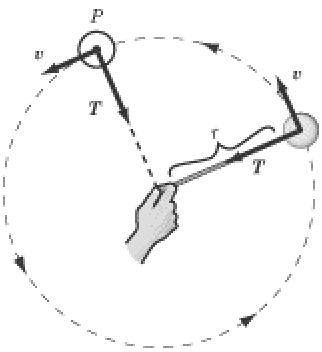
From kinematics, we know that acceleration is the rate of change of the velocity vector with time. If we consider two points very close together on the ball's trajectory and calculate $\Delta \mathbf{v}$, we find that the ball's acceleration points inward along the radius of the circle.



The acceleration of a body experiencing uniform circular motion is always directed toward the centre of the circle, so we call that acceleration **centripetal acceleration**, a_c. *Centripetal* comes from a Latin word meaning "centre-seeking." We define the centripetal acceleration of a body moving in a circle as:

$$a_c = \frac{v^2}{r}$$

Where ***v*** is the body's velocity and *r* is the radius of the circle. The body's centripetal acceleration is constant in magnitude but changes in direction. Note that even though the direction of the centripetal acceleration vector is changing, the vector always points toward the center of the circle.



How This Knowledge Will Be Tested

Most of us are accustomed to think of "change" as a change in magnitude, so it may be counterintuitive to think of the acceleration vector as "changing" when its magnitude remains constant. You'll frequently find questions on Physics that will try to catch you sleeping on the nature of centripetal acceleration. These questions are generally qualitative, so if you bear in mind that the acceleration vector is constant in magnitude, has a direction that always points toward the centre of the circle, and is always perpendicular to the velocity vector, you should have no problem at all.

Centripetal Force

Wherever you find acceleration, you will also find force. For a body to experience centripetal acceleration, a **centripetal force** must be applied to it. The vector for this force is similar to the acceleration vector: it is of constant magnitude, and always points radially inward to the center of the circle, perpendicular to the velocity vector. We can use Newton's Second Law and the equation for centripetal acceleration to write an equation for the centripetal force that maintains an object's circular motion:

$$\boldsymbol{F} = m\boldsymbol{a}_c = \frac{mv^2}{r}$$

Example

> A ball with a mass of 2 kg is swung in a circular path on a massless rope of length 0.5 m. If the ball's speed is 1 m/s, what is the tension in the rope?

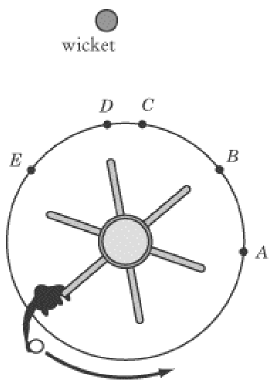
The tension in the rope is what provides the centripetal force, so we just need to calculate the centripetal force using the equation above:

$$\boldsymbol{T} = \frac{mv^2}{r} = \frac{(2\ \mathrm{kg})(1\ \mathrm{m/s})^2}{0.5\ \mathrm{m}} = 4\ \mathrm{N}$$

Objects Released from Circular Motion

One concept that is tested frequently on SAT II Physics is the trajectory of a circling body when the force providing centripetal acceleration suddenly vanishes. For example, imagine swinging a ball in a circle overhead and then letting it go. As soon as you let go, there is no longer a centripetal force acting on the ball. Recall Newton's First Law: when no net force is acting on an object, it will move with a constant velocity. When you let go of the ball, it will travel in a straight line with the velocity it had when you let go of it.

Example



A student is standing on a merry-go-round that is rotating counterclockwise, as illustrated above. The student is given a ball and told to release it in such a way that it knocks over the wicket at the top of the diagram. At what point should the student release the ball?

When the student releases the ball, it will travel in a straight line, tangent to the circle. In order to hit the wicket, then, the student should release the ball at point *B*.